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# ABSTRACT

In this paper the recently developed method, Grey Wolf Optimizer, is employed for design of castellated beams. These types of open-web beams have found widespread use, primarily in buildings, because of great savings in materials and construction costs. Hence, the minimum cost is taken as the design objective function and the Grey Wolf Optimizer method is utilized for obtaining the solution of the design problem. A number of design examples are selected from literature to demonstrate the efficiency of the utilized algorithm. The results demonstrate that the Grey Wolf Optimizer algorithm is a potential alternative optimization algorithm to solve castellated beam problems.

**Keywords:** Grey wolf optimizer (GWO); optimum design; castellated beams; hexagonal opening beam; circular opening beam.

# **1. INTRODUCTION**

Since the 1940's, the production of structural beams with higher strength and lower cost has been an asset to engineers in their efforts to design more efficient steel structures [1]. The castellated I-shaped steel beam is one of these efforts that have a widespread application in steel building construction especially in the form of simply supported main gravity girders.

In design of steel structures, beams with web-opening are widely used to pass the under floor services ducts such as water pipes and air ducts. Castellated beam is created from a standard wide-flange beam by cutting it longitudinally in a zig-zag or semi-circular pattern, separating and offsetting the two halves, and welding them back together. The resulting holes in the webs permit mechanical ducts, plumbing, and electrical lines to pass through the beam rather than beneath the beam [2-5].

In recent years, a great deal of progress has been made in the design of steel beams with web-openings and a cellular beam is one of them. A cellular beam is the modern form of the

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traditional castellated beam, but with a far wider range of applications for floor beams in particular. Cellular beams are steel sections with circular openings that are made by cutting a rolled beam web in a half circular pattern along its centerline and re-welding the two halves of hot rolled steel sections as shown in Fig. 1. This opening increases the overall beam depth; moment of inertia and section modulus, without increasing the overall weight of the beam [6].



Figure 1. Castellated beam with different web openings

Many meta-heuristic algorithms have been developed in the last two decades to help solving optimization problems that were previously difficult or impossible to solve using mathematical programming algorithms. In general, these algorithms are simple to implement, present (near) optimal solutions in acceptable computation times even in complex search spaces. There are different meta-heuristic optimization methods; Genetic Algorithms (GA) [7], Ant Colony Optimization (ACO) [8], Particle Swarm Optimizer (PSO) [9], Harmony Search algorithm (HS) [10], Charged System Search method (CSS) [11], Bat algorithm [12], Ray optimization algorithm (RO) [13], Krill-herd algorithm [14], Dolphin Echolocation algorithm (DE) [15], Colliding Bodies Optimization (CBO) [16] are some of such meta-heuristic algorithms. In this study, one of the recently developed algorithms called Grey Wolf Optimizer (GWO) [17] is used for optimal design of castellated beams. The great advantages of GWO are that the algorithm is simple, flexible and easy to implement. Also there are fewer control parameters to tune.

The main objective of this paper is to investigate the differences in cost associated with the castellated beams with hexagonal opening and cellular beams. Here, the GWO algorithm is utilized for optimization and cost of the beam is considered as the objective function.

According to the above-mentioned content, this paper is organized as follows: In Section 2, the design of castellated beam is introduced. Statement of the optimization design problem is formulated in Section 3. In Section 4, the GWO algorithm is briefly introduced. In Section 5, the cost of castellated beam as the design objective function is minimized, and finally Section 6 concludes the paper.

# 2. DESIGN OF CASTELLATED BEAMS

The performance of any beam is dependent upon the physical dimensions as well as the cross-section geometry and shape. The design concept for castellated beams is based on typical beam limit states, but the presence of web openings and welds can cause other modes of failure. The potential modes of failure associated with castellated beams are:

- 1. Vierendeel bending mechanism;
- 2. Lateral-torsional buckling;
- 3. Rupture of the welded joint;
- 4. Web post buckling due to shear force;
- 5. Compression web post buckling;
- 6. Flexural failure mechanism;

Lateral- torsional buckling may occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. In this paper it is assumed that the compression flange of the castellated beam is restrained by the floor system. Therefore, the overall buckling strength of the castellated beam is omitted from the design consideration. These modes are closely associated with beam geometry, shape parameters, type of loading, and provision of lateral supports. In the design of castellated beams, these criteria should be considered [18-24]:

### 2.1 Overall beam flexural capacity

Under applied load combinations the castellated beam should have sufficient flexural capacity to be able to resist the external loading. This is given by following expression:

$$M_U \le M_P = A_{LT} P_Y H_U \tag{1}$$

where  $A_{LT}$  is the area of lower tee,  $P_{Y}$  is the design strength of steel, and  $H_{U}$  is distance between center of gravities of upper and lower tees.

#### 2.2 Beam shear capacity

In the design of castellated beams, it is necessary to control two modes of shear failures. The first one is the vertical shear capacity and the upper and lower tees should undergo that. Sum of the shear capacity of the upper and lower tees are checked using the following equations:

$$P_{VY} = 0.6P_Y (A_{WUL})$$
 For circular opening  

$$P_{VY} = \frac{\sqrt{3}}{3} P_Y (A_{WUL})$$
 For hexagonal opening (2)

The second one is the horizontal shear capacity. It is developed in the web post due to the change in axial forces in the tee-section as shown in Fig. 2. Web post with too short mid-depth welded joints may fail prematurely when horizontal shear exceed the yield strength.

The horizontal shear capacity is checked using the following equations:

$$P_{VH} = 0.6P_Y (0.90A_{WP})$$
 For circular opening  

$$P_{VH} = \frac{\sqrt{3}}{3} P_Y (A_{WP})$$
 For hexagonal opening (3)

where  $A_{WUL}$  is the total area of the webs of the tees and  $A_{WP}$  is the minimum area of web post.



Figure 2. Horizontal shear in the web post of castellated beams, (a) hexagonal opening, (b) circular opening

### 2.3 Flexural and buckling strength of web post

As above mentioned, it is assumed that the compression flange of the castellated beam restrained by the floor system. Thus the overall buckling of the castellated beam is omitted from the design consideration. The web post flexural and buckling of capacity in castellated beam is given by:

$$\frac{M_{MAX}}{M_E} = [C_1 . \alpha - C_2 . \alpha^2 - C_3]$$
(4)

where  $M_{MAX}$  is the maximum allowable web post moment and  $M_E$  is the web post capacity at critical section A-A shown in Fig. 2.  $C_1, C_2$  and  $C_3$  are constants obtained by following expressions:

$$C_1 = 5.097 + 0.1464(\beta) - 0.00174(\beta)^2$$
(5)

$$C_2 = 1.441 + 0.0625(\beta) - 0.000683(\beta)^2$$
(6)

$$C_3 = 3.645 + 0.0853(\beta) - 0.00108(\beta)^2$$
<sup>(7)</sup>

where 
$$\alpha = \frac{S}{2d}$$
 for hexagonal openings, and  $\alpha = \frac{S}{D_0}$  for circular openings, also  $\beta = \frac{2d}{t_w}$  for

hexagonal openings, and  $\beta = \frac{D_0}{t_w}$  for circular openings, S is the spacing between the centers

of holes, d is the cutting depth of hexagonal opening,  $D_0$  is the holes diameter and  $t_w$  is the web thickness.

### 2.4 Vierendeel bending of upper and lower tees

Vierendeel mechanism is always critical in steel beams with web openings, where global shear force is transferred across the opening length, and the Vierendeel moment is resisted by the local moment resistances of the tee-sections above and below the web openings.

Vierendeel bending results in the formation of four plastic hinges above and below the web opening. The overall Vierendeel bending resistance depends on the local bending resistance of the web-flange sections. This mode of failure is associated with high shear forces acting on the beam. The Vierendeel bending stresses in the circular opening obtained by using the Olander's approach. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as follows [20]:

$$\frac{P_0}{P_U} + \frac{M}{M_P} \le 1.0$$
(8)

where  $P_0$  and M are the force and the bending moment on the section, respectively.  $P_U$  is equal to area of critical section  $\times P_Y$ ,  $M_P$  is calculated as the plastic modulus of critical section  $\times P_Y$  in plastic section or elastic section modulus of critical section  $\times P_Y$  for other sections.

The plastic moment capacity of the tee-sections in castellated beams with hexagonal opening are calculated independently. The total of the plastic moment is equal to the sum of the Vierendeel resistances of the above and below tee-sections. The interaction between Vierendeel moment and shear forces should be checked by the following expression:

$$V_{OMAX} \cdot e - 4M_{TP} \le 0 \tag{9}$$

where  $V_{OMAX}$  and  $M_{TP}$  are the maximum shear force and the moment capacity of teesection, respectively.

### 2.5 Deflection of castellated beam

Serviceability checks are high importance in the design, especially in beams with web opening where the deflection due to shear forces is significant. The deflection of a castellated beam under applied load combinations should not exceed span/360. In castellated beams with circular opening, the deflection at each point is calculated by following expression:

$$Y_{TOT} = Y_{MT} + Y_{WP} + Y_{AT} + Y_{ST} + Y_{SWP}$$
(10)

where  $Y_{MT}$ ,  $Y_{WP}$ ,  $Y_{AT}$ ,  $Y_{ST}$  and  $Y_{SWP}$  are deflection due to bending moment in tee, deflection due to bending moment in web post of beam, deflection due to axial force in tee, deflection due to shear in tee and deflection due to shear in web post, respectively. These equations are provided in Ref. [20].

For a castellated beam with hexagonal opening and length L subjected to transverse loading, the total deflection is composed by two terms: the first term corresponds to pure moment action  $f_b$ , and the second one corresponds to shear action  $f_s$ . Thus, the total deflection can be calculated by the following expression:

$$f = f_b + f_s = c_1 L^3 + c_2 L \tag{11}$$

 $c_1$  and  $c_2$  are determined by means of a curve fitting technique [22].

### **3. OPTIMIZATION OF CASTELLATED BEAMS**

The main goal for producing and using castellated beam is to suppress the cost of material by applying more efficient cross sectional shapes made from standard profiles in combination with aesthetic and architectural design considerations.

In a castellated beam, there are many factors that require special considerations when estimating the cost of beam, such as man-hours of fabrication, weight, price of web cutting and welding process. It is assumed that the costs associated with man-hours of fabrication for hexagonal and circular opening are identical. Thus, the objective function includes three parts: The beam weight, price of the cutting and price of the welding. The objective function can be expressed as:

$$F_{\text{cost}} = \rho A_{\text{initail}}(L_0) P_1 + L_{cut} P_2 + L_{weld} P_3$$
(12)

where  $P_1$ ,  $P_2$  and  $P_3$  are the price of the weight of the beam per unit weight, length of cutting and welding for per unit length,  $L_0$  is the initial length of the beam before castellation process,  $\rho$  is the density of steel,  $A_{\text{initial}}$  is the area of the selected universal beam section,  $L_{cut}$  and  $L_{weld}$  are the cutting length and welding length, respectively. The length of cutting

is different for hexagonal and circular web-openings. The dimension of the cutting length is described by following equations:

For circular opening,

$$L_{\rm cut} = \pi D_0 .NH + 2e (NH + 1) + \frac{\pi D_0}{2} + e$$
(13)

For hexagonal opening,

$$L_{cut} = 2NH\left(e + \frac{d}{\sin(\theta)}\right) + 2e + \frac{d}{\sin(\theta)}$$
(14)

where *NH* is the total number of holes, *e* is the length of horizontal cutting of web,  $D_0$  is the diameter of holes, *d* is the cutting depth, and  $\theta$  is the cutting angle.

Also, the welding length for both of circular and hexagonal openings is determined by Eq. (15).

$$L_{weld} = e(NH+1) \tag{15}$$

### 3.1 Design of castellated beam with circular opening

Design process of a cellular beam consists of three phases: The selection of a rolled beam, the selection of a diameter, and the spacing between the center of holes or total number of holes in the beam as shown in Fig. 1, [20, 21]. Hence, the sequence number of the rolled beam section in the standard steel sections tables, the circular holes diameter and the total number of holes are taken as design variables in the optimum design problem. The optimum design problem formulated by considering the constraints explained in the previous sections can be expressed as the following:

Find an integer design vector  $\{X\} = \{x_1, x_2, x_3\}^T$  where  $x_1$  is the sequence number of the rolled steel profile in the standard steel section list,  $x_2$  is the sequence number for the hole diameter which contains various diameter values, and  $x_3$  is the total number of holes for the cellular beam [20]. Hence the design problem can be expressed as:

Minimize Eq. (12) Subjected to

$$g_1 = 1.08 \times D_0 - S \le 0 \tag{16}$$

$$g_2 = S - 1.60 \times D_0 \le 0 \tag{17}$$

$$g_3 = 1.25 \times D_0 - H_s \le 0 \tag{18}$$

$$g_4 = H_s - 1.75 \times D_0 \le 0 \tag{19}$$

$$g_5 = M_U - M_P \le 0 \tag{20}$$

$$g_6 = V_{MAXSUP} - P_V \le 0 \tag{21}$$

$$g_7 = V_{OMAX} - P_{VY} \le 0 \tag{22}$$

$$g_8 = V_{HMAX} - P_{VH} \le 0$$
(23)  
$$= M - M \le 0$$
(24)

$$a_{0} = V_{A-AMAX} + N_{WMAX} = 0$$
(24)  
$$a_{0} = V_{A-AMAX} + 0$$
(25)

$$\begin{array}{ccc} g_{10} - v_{TEE} & -0.50 \times I_{VY} \leq 0 \\ P_0 & M & 1.0 \leq 0 \end{array} \tag{25}$$

$$g_{11} = \frac{I_0}{P_U} + \frac{M}{M_P} - 1.0 \le 0 \tag{26}$$

$$g_{12} = Y_{MAX} - \frac{L}{360} \le 0 \tag{27}$$

where  $t_W$  is the web thickness,  $H_S$  and L are the overall depth and the span of the cellular beam, and S is the distance between centers of holes.  $M_U$  is the maximum moment under the applied loading,  $M_P$  is the plastic moment capacity of the cellular beam,  $V_{MAXSAP}$  is the maximum shear at support,  $V_{OMAX}$  is the maximum shear at the opening,  $V_{HMAX}$  is the maximum horizontal shear,  $M_{A-AMAX}$  is the maximum moment at A-A section shown in Figure. 2.  $M_{WMAX}$  is the maximum allowable web post moment,  $V_{TEE}$  represent the vertical shear on the tee at  $\theta = 0$  of web opening,  $P_0$  and M are the internal forces on the web section, and  $Y_{MAX}$  denotes the maximum deflection of the cellular beam [20, 24].

## 3.2 Design of castellated beam with hexagonal opening

In design of castellated beams with hexagonal openings, the design vector includes four design variables: The selection of a rolled beam, the selection of a cutting depth, the spacing between the center of holes or total number of holes in the beam and the cutting angle as shown in Fig. 2. Hence the optimum design problem formulated by the following expression:

Find an integer design vector  $\{X\} = \{x_1, x_2, x_3, x_4\}^T$  where  $x_1$  is the sequence number of the rolled steel profile in the standard steel section list,  $x_2$  is the sequence number for the cutting depth which contains various values,  $x_3$  is the total number of holes for the castellated beam and  $x_4$  is the cutting angle. So, the design problem turns out to be as follows:

Minimize Eq. (12) Subjected to

$$g_1 = d - \frac{3}{8} (H_s - 2t_f) \le 0$$
<sup>(28)</sup>

$$g_2 = (H_s - 2t_f) - 10 \times (d_T - t_f) \le 0$$
<sup>(29)</sup>

$$g_3 = \frac{2}{3} \cdot d \cdot \cot \phi - e \le 0 \tag{30}$$

$$g_4 = e - 2d \cot \phi \le 0 \tag{31}$$

$$g_5 = 2d \cot \phi + e - 2d \le 0 \tag{32}$$

$$g_6 = 45^{\circ} - \phi \le 0 \tag{33}$$

$$g_7 = \phi - 64^\circ \le 0 \tag{34}$$

$$g_8 = M_U - M_P \le 0 \tag{35}$$

$$g_9 = V_{MAXSUP} - P_V \le 0 \tag{36}$$

$$g_{10} = V_{OMAX} - P_{VY} \le 0 \tag{37}$$

$$g_{11} = V_{HMAX} - P_{VH} \le 0 \tag{38}$$

$$g_{12} = M_{A-AMAX} - M_{WMAX} \le 0 \tag{39}$$

$$g_{13} = V_{TEE} - 0.50 \times P_{VY} \le 0 \tag{40}$$

$$g_{14} = V_{OMAX} \cdot e - 4M_{TP} \le 0 \tag{41}$$

$$g_{15} = Y_{MAX} - \frac{L}{360} \le 0 \tag{42}$$

where  $t_f$  is the flange thickness,  $d_T$  is the depth of the tee-section,  $M_P$  is the plastic moment capacity of the castellated beam,  $M_{A-AMAX}$  is the maximum moment at A-A section shown in Fig. 2,  $M_{WMAX}$  is the maximum allowable web post moment,  $V_{TEE}$  represent the vertical shear on the tee,  $M_{TP}$  is the moment capacity of tee-section and  $Y_{MAX}$  denotes the maximum deflection of the castellated beam with hexagonal opening [2].

# 4. GREY WOLF OPTIMIZER

In recent years, meta-heuristic optimization algorithms have become very popular and widely used to solve many problems from different fields. The meta-heuristics are inspired from nature, typically related to physical phenomena, animal's behaviors, or evolutionary concepts.

Grey Wolf Optimizer (GWO) is one of the recent meta-heuristic algorithms developed by Mirjalili *et al.* in 2014 [17]. This method mimics the hunting behavior and the social hierarchy of grey wolves. The grey wolves are categorized according to societal hierarchy as alpha, beta, delta and omega. The alphas are the dominant and the group follows his/her instructions and the betas, the secondary wolves that assist the alphas in making decisions. Omega is the lowest ranking grey wolves. If a wolf is neither an alpha nor a beta, or an omega, he/she is called delta. Delta wolves come in the hierarchy next to the alphas and betas but they lead the omega. A summary of this method is described in the following subsections.

### 4.1 Social hierarchy

Grey wolves mostly prefer to live in a group. They have a strict social dominant hierarchy and the fitness solutions are structured according to this feature of wolves. The best fitness solution is regarded as alpha ( $\alpha$ ) followed by beta( $\beta$ ), delta ( $\delta$ ) and omega ( $\omega$ ) wolves.

#### 4.2 Encircling prey

In addition to the social hierarchy of grey wolves, group hunting is another interesting social behavior of grey wolves. A grey wolf can update its position inside the space around the prey in any random location by using Eqs. (43) and (44).

$$\vec{D} = \left| \vec{C}.\vec{X}_{p}(t) - \vec{X}(t) \right| \tag{43}$$

$$\vec{X}(t+1) = \vec{X}_{p}(t) - \vec{A}.\vec{D}$$
(44)

where *t* indicates the current iteration;  $\vec{A}$  and  $\vec{C}$  are coefficient vectors;  $\vec{X}_p$  is the position vector of the prey; and  $\vec{X}$  indicates the position vector of a grey wolf. The coefficients  $\vec{A}$  and  $\vec{C}$  are computed using the following equations:

$$\vec{A} = 2\vec{a}.\vec{r_1} - \vec{a} \tag{45}$$

$$\vec{C} = 2.\vec{r}_2 \tag{46}$$

where  $\vec{r_1}$  and  $\vec{r_2}$  are random values in [0,1]; The components of  $\vec{a}$  are linearly decreased from 2 to 0 over the course of iterations.

### 4.3 Search for prey (Exploration)

Grey wolves mostly search according to the position of the alpha, beta, and delta. They diverge from each other to search for prey and converge to attack prey. In order to mathematically model divergence,  $\vec{A}$  is utilized with random values greater than 1 or less than -1 to oblige the search agent to diverge from the prey. This emphasizes exploration and allows GWO to search globally.

Another component of GWO that favors exploration is  $\vec{C}$ . The  $\vec{C}$  vector contains random values in [0, 2]. This component provides random weights for prey in order to stochastically emphasize (C > 1) or deemphasize (C < 1) the effect of prey in defining the distance in Eq. (43). This assists GWO to show a more random behavior throughout optimization, favoring exploration and local optima avoidance. It is worth mentioning that  $\vec{C}$  is not linearly decreased in contrast to  $\vec{A}$ . GWO deliberately requires  $\vec{C}$  providing random values at all times to emphasize exploration/exploitation not only during initial iterations but also final iterations. This component is very helpful in case of local optima stagnation, especially in the final iterations.

### 4.4 Attacking prey (Exploitation)

In this phase, the value of  $\vec{a}$  is reduced and thereby the fluctuation range of  $\vec{A}$  is decreased. When  $\vec{A}$  has random values in the range [-1, 1], then the search agent's next location will be in any place between its current position and the position of the prey.

### 4.5 Hunting

The alpha, beta and delta type grey wolves are the best solution and they have superior knowledge about the potential location of prey. Hence, the first three best solutions acquired are saved and force the other search agents to update their positions according to the location of the best search agents [17]. The following equations can be used for this purpose:

$$\vec{D}_{\alpha} = \left| \vec{C}_{1} \vec{X}_{\alpha} - \vec{X} \right| \tag{47}$$

$$\vec{D}_{\beta} = \left| \vec{C}_2 \vec{X}_{\beta} - \vec{X} \right| \tag{48}$$

$$\vec{D}_{\delta} = \left| \vec{C}_{3} \vec{X}_{\delta} - \vec{X} \right| \tag{49}$$

$$\vec{X}_{1} = \vec{X}_{\alpha} - \vec{A}_{1} \cdot (\vec{D}_{\alpha})$$
(50)

$$\vec{X}_{2} = \vec{X}_{\beta} - \vec{A}_{2}.(\vec{D}_{\beta})$$
 (51)

$$\vec{X}_{3} = \vec{X}_{\delta} - \vec{A}_{3} \cdot (\vec{D}_{\delta})$$
(52)

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}$$
(53)

## **5. DESIGN EXAMPLES**

In this section, in order to compare fabrication cost of the castellated beams with circular and hexagonal holes, three beams are solved using GWO method. Among the steel section list of British Standards 64 Universal Beam (UB) sections starting from  $254 \times 102 \times 28$  UB to  $914 \times 419 \times 388$  UB are chosen to constitute the discrete set for steel sections from which the design algorithm selects the sectional designations for the castellated beams. In the design pool of holes diameters 421 values are arranged which varies between 180 and 600 mm with increment of 1 mm. Also, for cutting depth of hexagonal opening, 351 values are considered which varies between 50 and 400 mm with increment of 1 mm and cutting angle changes from 45 to 64. Another discrete set is arranged for the number of holes. Likewise, in all the design problems, the modulus of elasticity is equal to 205 kN/mm<sup>2</sup> and Grade 50 is selected for the steel of the beam which has the design strength 355 MPa. The coefficients  $P_1$ ,  $P_2$  and  $P_3$  in the objective function are considered 0.85, 0.30 and 1.00, respectively.

### 5.1 Castellated beam with 4-m span

A simply supported beam with 4m span shown in Fig. 3 is selected as the first design

example. The beam is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 kN also acts at mid-span of the beam and the allowable displacement of the beam is limited to 12 mm. The number of CBs is taken as 50 and maximum number of iterations is considered 200.



Figure 3. Simply supported beam with 4m span

Beams with hexagonal and circular openings are separately designed by using of GWO algorithm. The optimum results obtained by this method and other meta-heuristics [25-28] are given in Table 1. The minimum cost for castellated beam with hexagonal hole which obtained by GWO, is equal to 89.78\$ while its value for cellular beam is equal to 91.15\$. These results indicate that the castellated beam with hexagonal opening have less cost in comparing with the cellular beam. Fig. 4 shows the convergence of GWO algorithm for design of castellated beams with different openings.

Table 1: Optimum designs of the castellated beams with 4m span						
Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost (\$)	Type of the hole
CBO-PSO algorithm [27]	UB 305×102×25	125	14	57	89.78	
CBO algorithm [26]	UB 305×102×25	125	14	57	89.78	Hevagonal
ECSS algorithm [25] GWO(present work)	UB 305×102×25 UB 305×102×25	125 125	14 14	57 57	89.78 89.78	Tiexagonai
CBO-PSO algorithm [27]	UB 305×102×25	243	14	_	91.08	
CBO algorithm [26]	UB 305×102×25	244	14	_	91.14	Circular
ECSS algorithm [25] GWO(present work)	UB 305×102×25 UB 305×102×25	248 249	14 14	_	96.32 91.15	encular



Figure 4. Convergence history of the castellated beams with 4m span

### 5.2 Castellated beam with 8m span

In the second example the GWO algorithm is used to design a simply supported castellated beam with a span of 8m. The beam carries a uniform dead load 0.40 kN/m, which includes its own weight. In addition, it is subjected to two concentrated loads; dead load of 70 kN and live load of 70 kN as shown in Fig. 5. The allowable displacement of the beam is limited to 23 mm. The number of CBs is taken as 50. The maximum number of iterations is considered 200.



Figure 5. A simply supported beam with 8m span

This beam is designed by several optimization methods. From Table 2, it can be seen that in design of the beam with hexagonal hole, the GWO has good performance in compression with other methods and the corresponding cost obtained by this method is equal to 718.20\$. Also, according to the obtained results, the designed beam with hexagonal opening in comparison with the cellular beam has less cost, and it is a more appropriate option in this case. The maximum value of the strength ratio is equal to 0.99 for both hexagonal and circular beams, and it is show that these constraints are dominant in the design of such beams. The convergence history for optimum design of castellated beam which is obtained by GWO is shown in Fig. 6.

A. Kaveh and F. Shokohi

Method	Optimum UB section	Hole diameter or cutting depth(mm)	Total number of holes	Cutting angle	Minimum cost (\$)	Type of the hole
CBO-PSO algorithm [27]	UB 610×229×101	244	14	55	718.33	
CBO algorithm [26]	UB 610×229×101	243	14	59	718.93	Havaganal
ECSS algorithm						nexagonai
[25]	UB 610×229×101	246	14	59	719.47	
GWO(present work)	UB 610×229×101	242	14	56	718.20	
CBO-PSO algorithm [27]	UB 610×229×101	487	14	_	721.55	
CBO algorithm [26]	UB 610×229×101	487	14	_	721.55	Circular
ECSS algorithm						Circulai
[25]	UB 610×229×101	487	14	_	721.55	
GWO(present	UB 610×229×101	487	14	_	721.58	
work)						

Table 2: Optimum designs of the castellated beams with 8m span



Figure 6. Convergence history of the castellated beams with 8m span

# 5.3 Castellated beam with 9m span

The beam with 9m span is considered as the last example of this study in order to compare the minimum cost of the castellated beams. The beam caries a uniform load of 40 kN/m including its own weight and two concentrated loads of 50 kN as shown in Fig. 7. The allowable displacement of the beam is limited to 25 mm. Similar to the two previous examples the number of CBs is taken as 50 and the maximum number of iterations is considered 200.



Figure 7. Simply supported beam with 9m span

Table 3 compares the results obtained by the GWO with those of the other algorithms. In the optimum design of castellated beam with hexagonal hole, GWO algorithm selects  $684 \times 254 \times 125$  UB profile, 16 holes, and 229 mm for the cutting depth and 57 for the cutting angle. The minimum cost of design is equal to 990.76\$. Also, in the optimum design of cellular beam, the GWO algorithm selects  $610 \times 229 \times 125$  UB profile, 15 holes and 489 mm for the holes diameter. Although the optimum value is obtained by CBO-PSO method, it can be seen that the GWO algorithm had good performance in this problem. The optimum shapes of the hexagonal and circular openings beams are illustrated as shown in Fig. 8.

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost (\$)	Type of the hole
CBO-PSO algorithm [27]	UB 684×254×125	230	16	56	990.33	
CBO algorithm [26]	UB 684×254×125	233	15	64	993.79	Havaganal
ECSS algorithm						Tiexagoliai
[25]	UB 684×254×125	277	13	56	995.97	
GWO(present work)	UB 684×254×125	229	16	57	990.76	
CBO-PSO algorithm [27]	UB 684×254×125	538	14	_	998.58	
CBO algorithm [26]	UB 684×254×125	538	14	_	997.75	Circular
ECSS algorithm						Circular
[25]	UB 684×254×125	539	14	_	998.94	
GWO(present work)	UB 610×229×125	489	15	-	995.89	

Table 3: Optimum designs of the castellated beams with 9m span



Figure 8. Optimum profiles of the castellated beams with hexagonal and circular openings for beam with 9m span

# 6. CONCLUDING REMARKS

In this study, the recently developed algorithm called GWO is used for optimum design of castellated beams. Three castellated beams are selected from literature to design by this method. Beams with hexagonal and circular openings are considered as web-opening of castellated beams. Also, the cost of the beam is considered as the objective function. Comparing the results obtained by GWO with other optimization methods demonstrates that GWO is a potential alternative optimization algorithm to solve castellated beam problems. It is observed that the optimization results obtained by the GWO algorithm have little difference with other methods for more design examples. The great advantages of the GWO are that the algorithm is simple, flexible, and easy to implement. Also there are fewer control parameters to tune. From the results obtained in this paper, it can be concluded that the use of the beam with hexagonal opening can lead to the use of less steel material and it is better than cellular beam from the point of view of the cost.

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